chapter (3)

Fluid statics

① Buoyancy and Floatation stability.

② Fluid Masses subjected to acceleration.

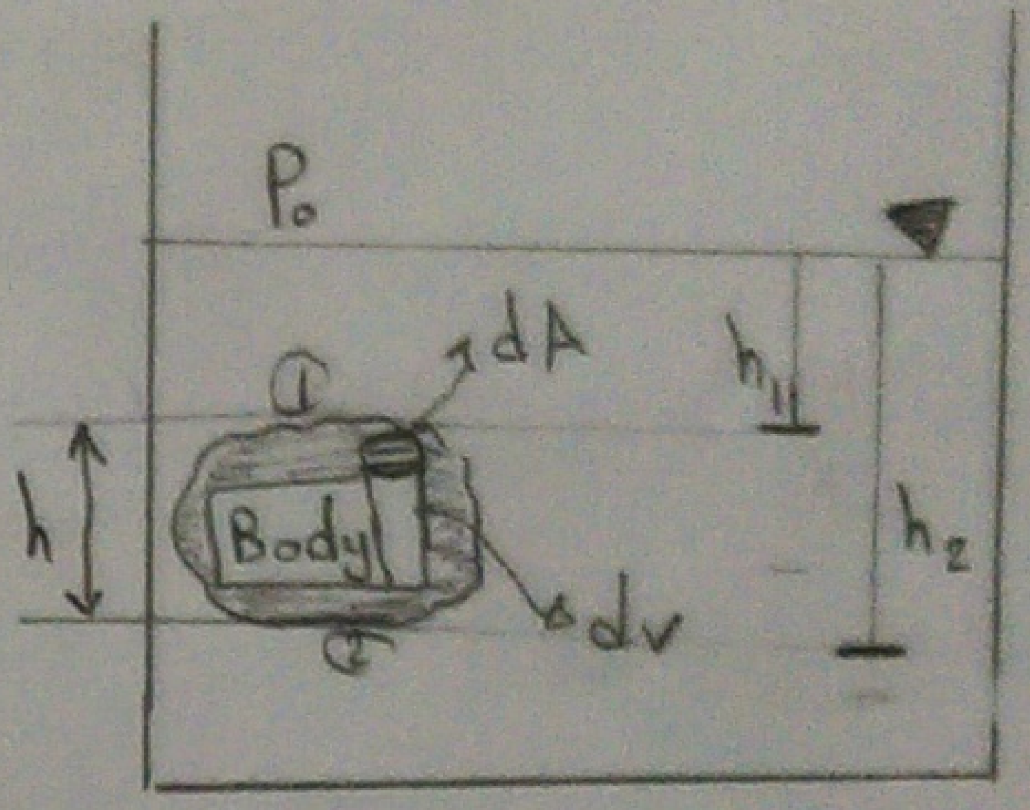
Buoyancy Principle

الجزء المغمور

Buoyancy Force $= (F_B) = W$ [submerged Body]

قوة الطفو

$$\forall = \int_{\forall} d\forall = \int_{A} h \, dA$$



$$\sum F_z = 0 = dF_2 - dF_1$$

$$= [(P_o + h_2 \rho g) dA] - [(P_o + h_1 \rho g) dA]$$

$F_1 = P_1 A \leftarrow P_1 = P_o + h_1 \rho g$

$F_2 = P_2 A \leftarrow P_2 = P_o + h_2 \rho g$

$$= h_2 \rho g dA - h_1 \rho g dA$$

$$= (h_2 - h_1) \rho g dA = \underbrace{h \rho g dA}_{\forall}$$

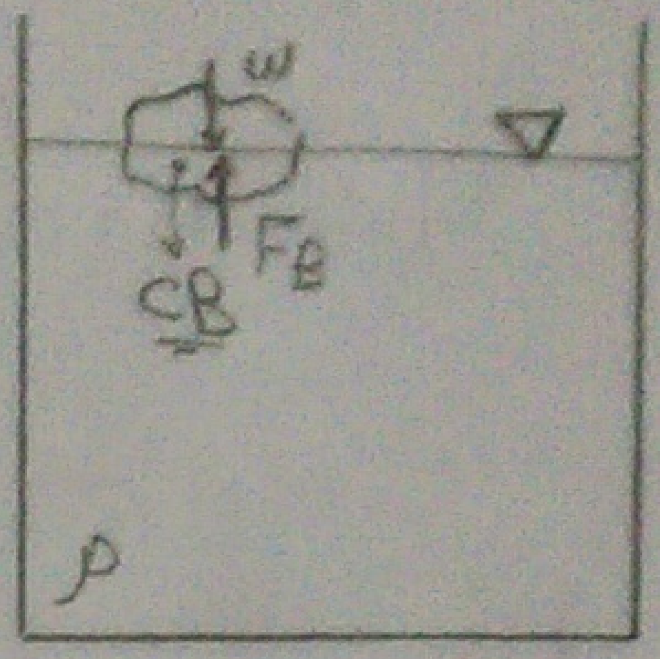
$$= \rho g \forall = m \cdot g = W \downarrow \uparrow F_B$$

Floating

$$F_B = w$$

$$\rho_F \, g \, \forall_F = \rho_{body} * g * \forall_{body}.$$

$\uparrow$ $\rho_F \, g \, \forall$ (Fraction of Body)

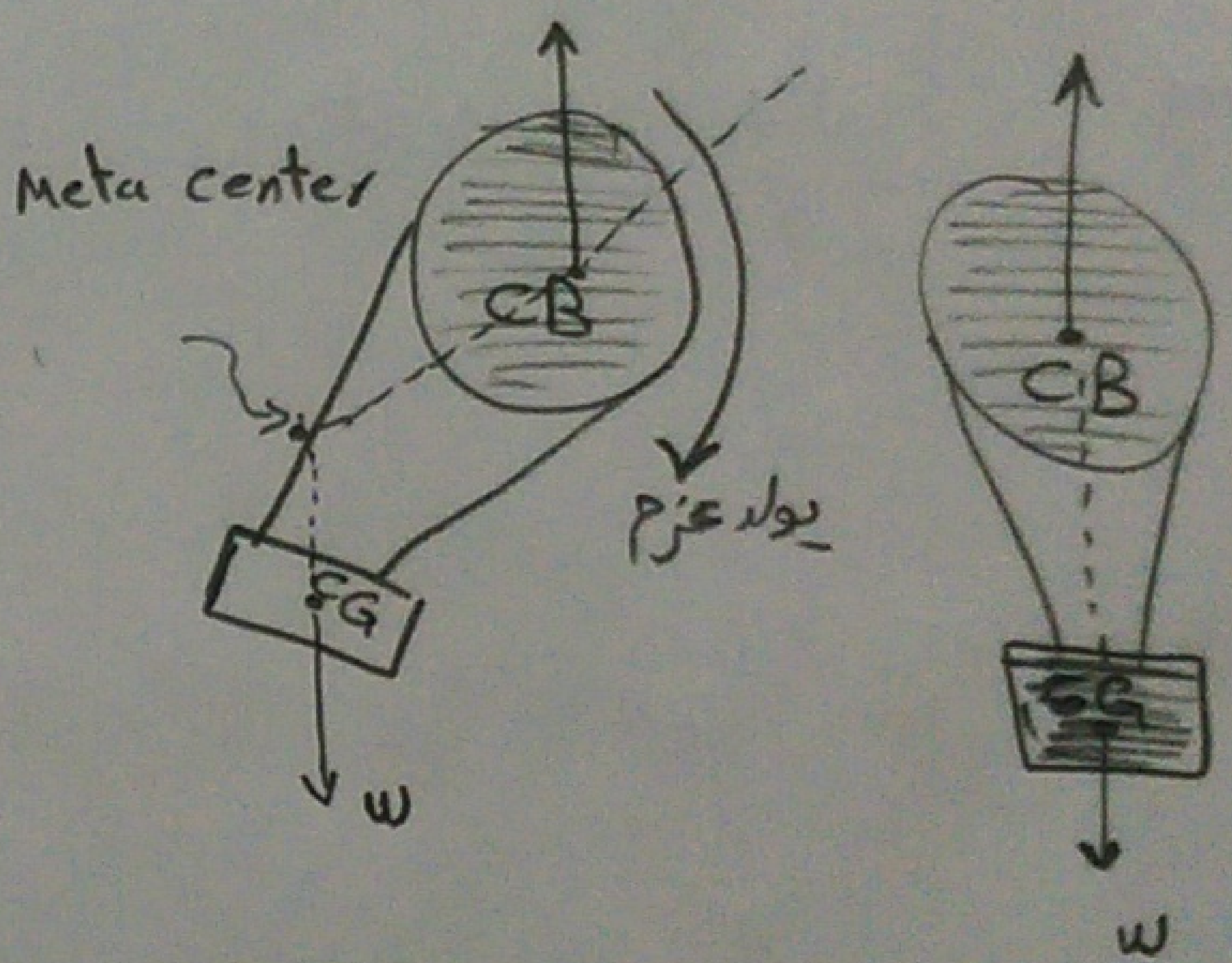


Stability of submerged Bodies

مغمور

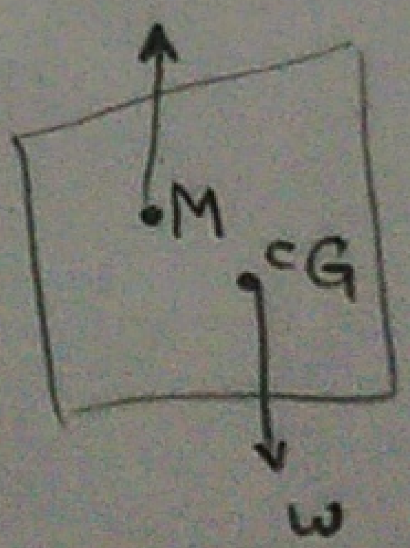
Meta center (مركز التأرجح)

هو ناتج من تقاطع خطين اتجاه الأصلي لقوة الدفع مع اتجاه الوزن للجسم .



$$M_B > M_G$$

M up CG



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Fluid (Rigid) Masses subject to acceleration

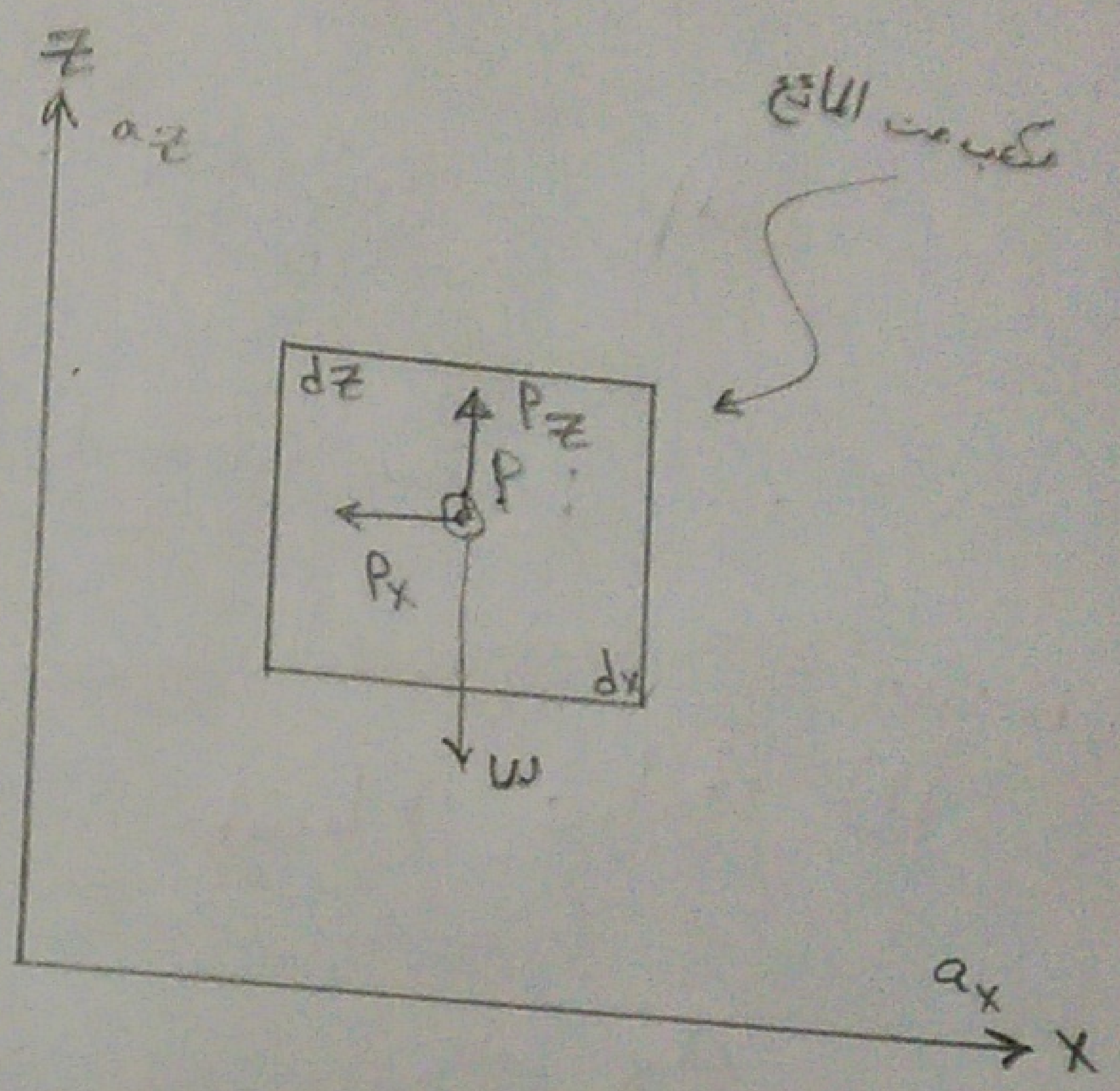
$W(\text{element}) = \gamma \cdot \rho \cdot g \cdot \forall$

$= \gamma \, dx \, dz$

الضغط يعمل كلا خرجيات النقطة (P).

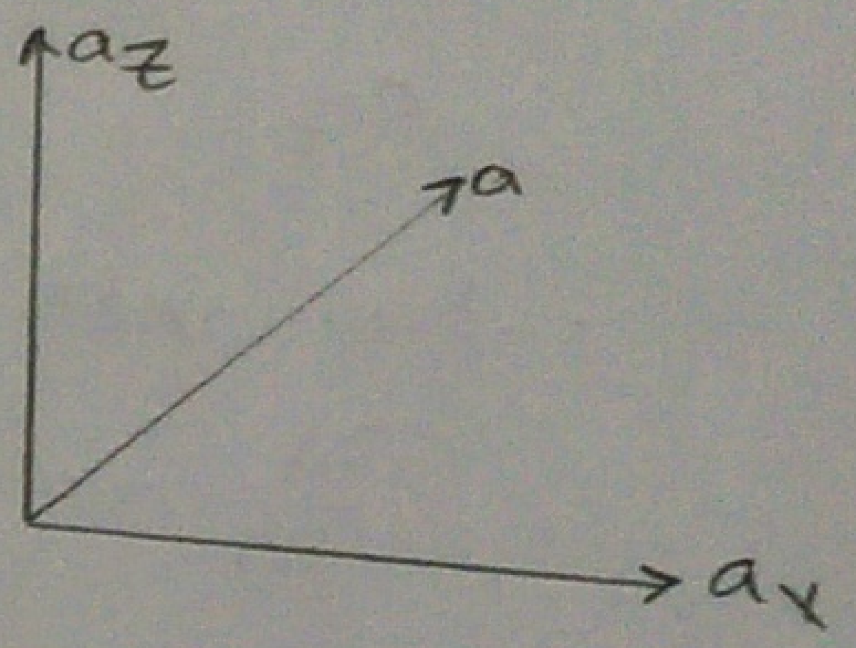
$$P_x = \frac{-\partial P}{\partial x}$$

$$P_z = \frac{-\partial P}{\partial z}$$



$$dm = \frac{\gamma}{g} \cdot dx \cdot dz$$

$$dF = dm \cdot a$$



$$dF_x = dm \cdot a_x$$

$$\left(\frac{-\partial P}{\partial x}\right) dx\,dz = \left(\frac{\gamma}{g} dx\,dz\right) \cdot a_x$$

$$\frac{\partial P}{\partial x} = \frac{-\gamma}{g} a_x = -\rho a_x$$

$$dF_z = dm * a_z$$

$$\frac{-\partial P}{\partial z} dx\,dz - \gamma\,dx\,dz = \left(\frac{\gamma}{g} dx\,dz\right) a_z$$

$$\frac{\partial P}{\partial z} = \frac{-\gamma}{g}(a_z + g)$$

d

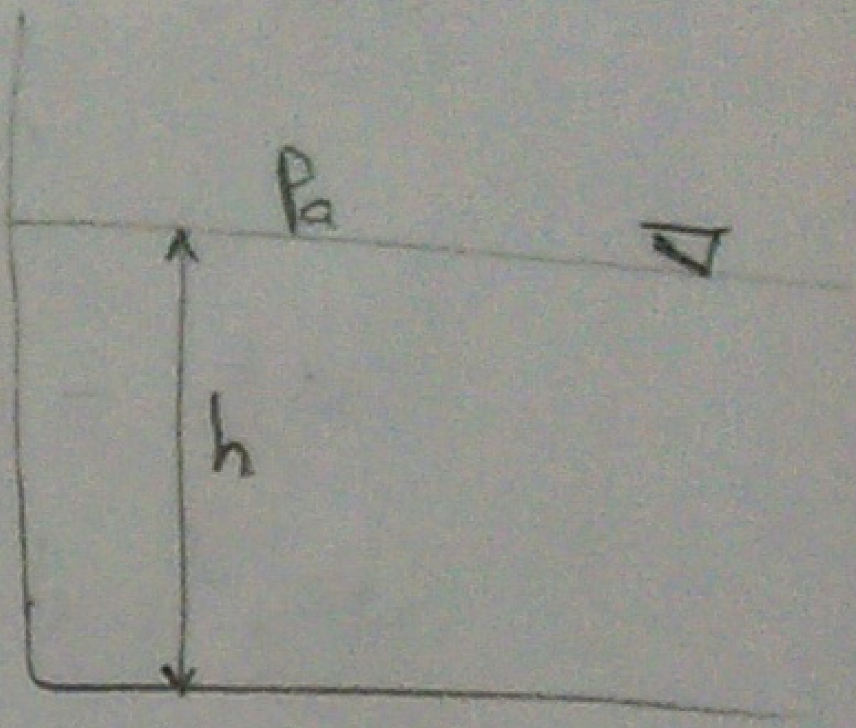
$$dp = \frac{\partial P}{\partial x} dx + \frac{\partial P}{\partial z} dz$$

$$dp = \frac{-\gamma}{g} a_x\, dx - \left[\frac{\gamma}{g}(a_z + g).dz\right]$$

Pressure gradient

حالات خاصة

1 static Fluid



$a_x = 0$

$a_z = 0$

بالتعويض في المعادلات

$$\frac{\partial P}{\partial x} = 0 \qquad \frac{\partial P}{\partial z} = -\gamma$$

$$\therefore P = \rho g h$$

$$\therefore \frac{dP}{dZ} = -\gamma$$

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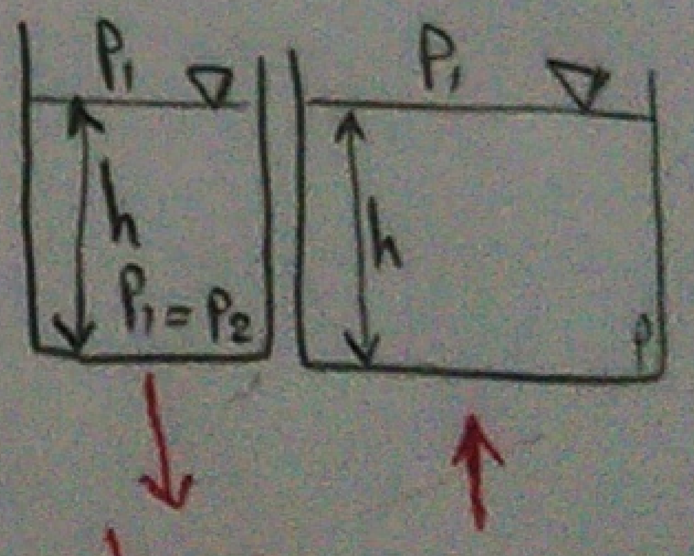
2 Free Falling of FLuid Mass

في حالة الهبوط

Falling



$a_x = 0$

$a_z = -g$

$$\frac{dp}{dz} = \frac{-\gamma}{g}(-g + g) = 0$$

في حالة السقوط الحر للموائع

$$\therefore P_2 = P_1$$

$a = -g$

$$-\frac{\partial P}{\partial z} = \frac{\partial P}{\partial x} = 0$$

في حالة الصعود

Going up

في حالة الصعود بعجلة الجاذبية
← الضغط يتضاعف.

$a_x = 0$

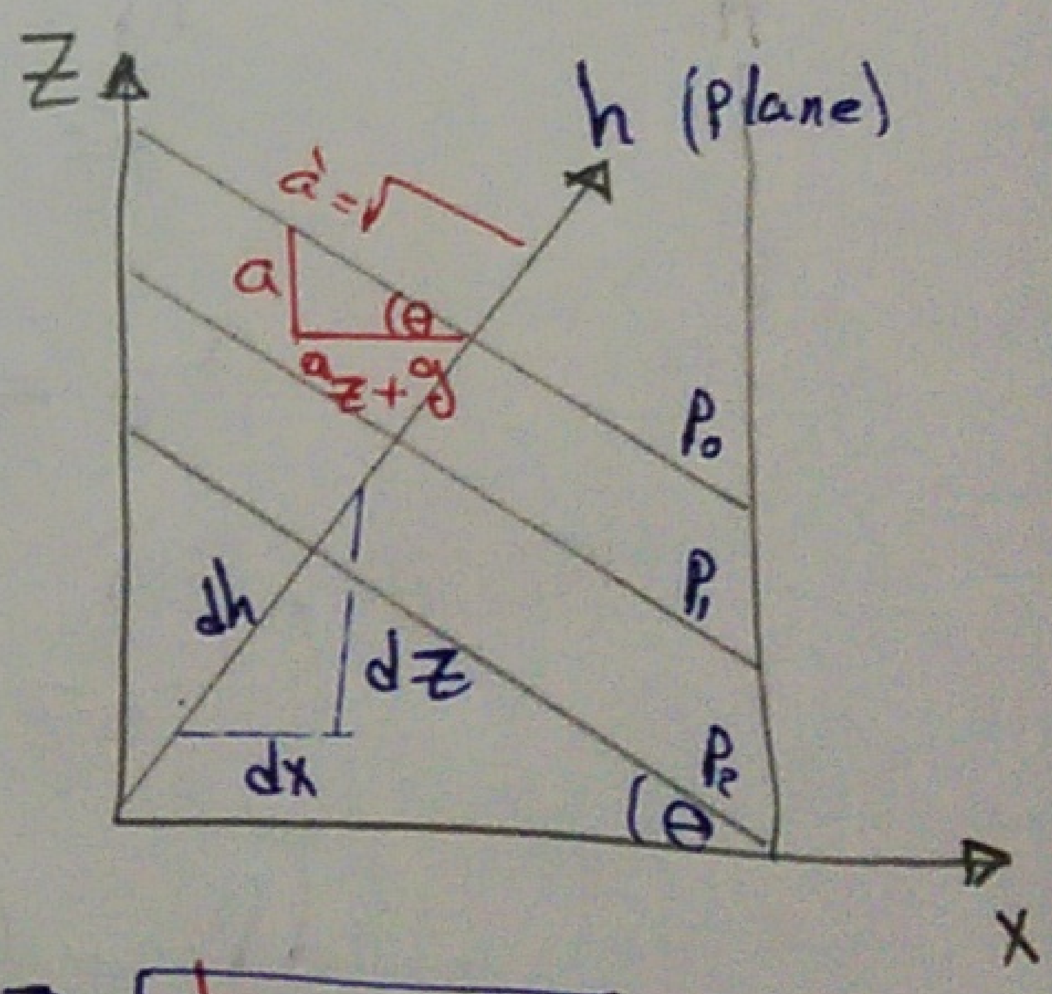
$a_z = g$

$$\frac{dP}{dz} = \frac{-\gamma}{g}\overset{2g}{(g+g)} = -2\gamma$$

$$\therefore P_2 = P_1 + 2\gamma h$$

3) For constant linear acceleration [inclinead plane]

مائع محفوظ في إناء ويتحرك على سطح مائل



a_x , a_z

$$\frac{dP}{dh} = \left(\frac{\partial P}{\partial x}\frac{dx}{dh}\right) + \left(\frac{\partial P}{\partial z}\frac{dz}{dh}\right)$$

$$\frac{dP}{dh} = -\gamma\left[\left(\frac{a_x}{g}\frac{dx}{dh}\right) + \left(\frac{a_z+g}{g}\frac{dz}{dh}\right)\right]$$

$$\frac{dx}{dh} = \frac{a_x}{a'}$$

$$\frac{dz}{dh} = \frac{a_z+g}{a'}$$

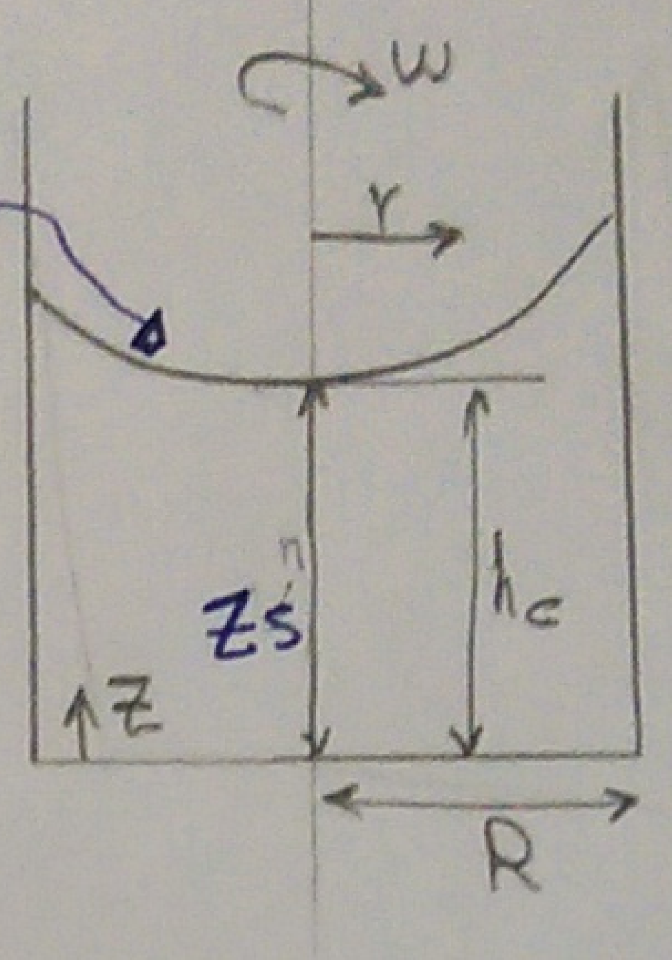
$$\frac{dP}{dh} = -\gamma\left(\frac{a'}{g}\right).$$

F

4) For Centrifugal (vortex) Motion :-

(r, θ, z) ← نستخدم الإحداثيات الإسطوانية

→ في تلك الحالة بـ Zero



$a_r = -w^2 r$

← لأنها خارجة من المركز.

$$\frac{\partial P}{\partial r} = \rho w^2 r = \frac{\gamma}{g} w^2 r$$

$$\frac{\partial P}{\partial z} =$$

$$a_z = 0$$

$$\frac{\partial P}{\partial z} = \frac{-\gamma}{g} g = -\gamma$$

$$\therefore dP = \left(\frac{\partial P}{\partial r}\right) dr + \left(\frac{\partial P}{\partial z}\right) dz$$

بالتعويض →

$$\boxed{dP = \rho r w^2 dr - \gamma dz}$$

$$\therefore dP = 0 \rightarrow \frac{dz}{dr} = r\frac{w^2}{g} \quad \text{For } dz = 0 \text{ (isobaric)}$$

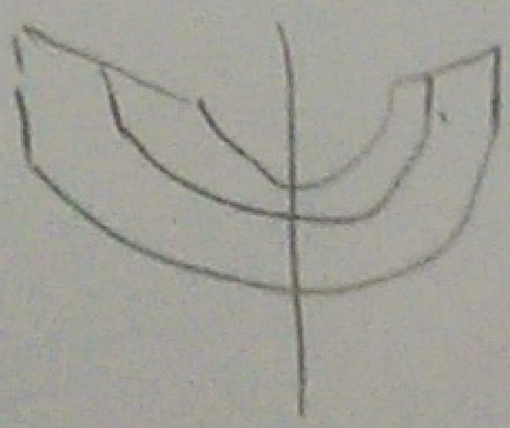
$$\therefore \; dz = \frac{r w^2}{g} \, dr$$

$$Z = \frac{r^2 w^2}{2g} + c$$

مح

سبب وجود شكل القطع المكافئ

← r^2



← c تدل على وجود مجموعة من المنحنيات

$$Z_s = h_c$$

ص ١٦٤ و ص ١٦٥

محاضرة النهاردة .